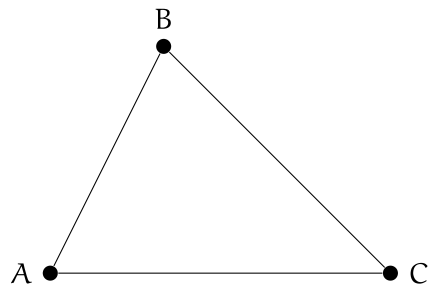
Chapter 1

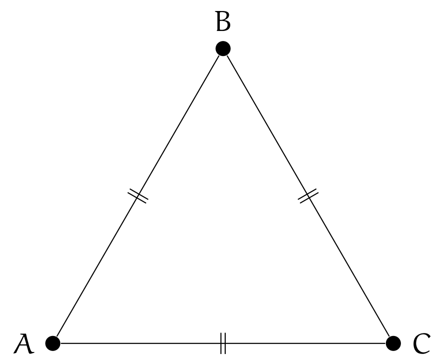
Introduction to Triangles

Connecting any three points with three line segments will get you a triangle. Here is the triangle ABC, which was created by connecting three points A, B, and C:

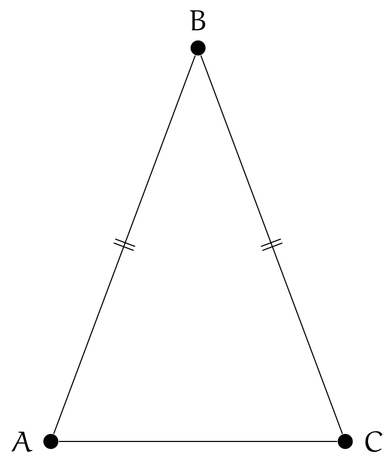


1.1 Equilateral, Isosceles, and Scalene Triangles

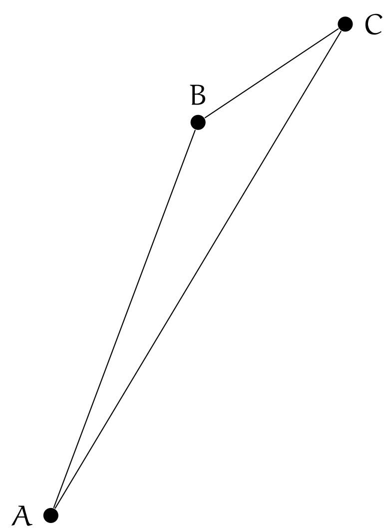
We talk a great deal about the length of the sides of triangles. If all three sides of the triangle are the same length, we say it is an *equilateral triangle*:



If only two sides of the triangle are the same length, we say it is an *isosceles triangle*:

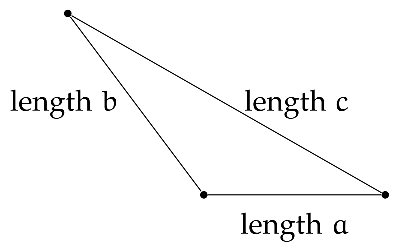


When all three sides of a triangle are different, the triangle is referred to as *scalene*. This consequently means all angle measures are different as well.



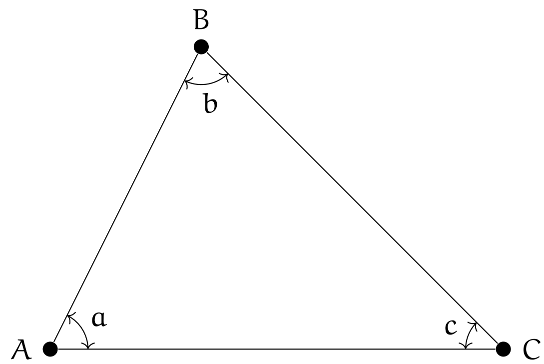
The shortest distance between two points is always the straight line between them. This means you can be certain that the length of one side will *always* be less than the sum of the lengths of the remaining two sides. This is known as the *triangle inequality*.

For example, in this diagram, c must be less than $a + b$.

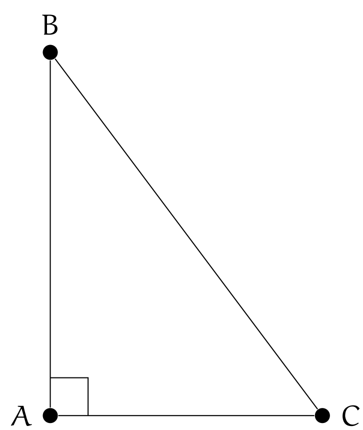


1.2 Interior Angles of a Triangle

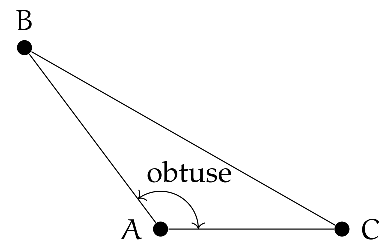
We also talk a lot about the interior angles of a triangle:



A triangle where one of the interior angles is a right angle is said to be a *right triangle*: The side opposite the right angle is the longest side, referred to as the hypotenuse.



If a triangle has an obtuse interior angle, it is said to be an *obtuse triangle*:



If all three interior angles of a triangle are less than 90°, it is said to be an *acute triangle*.

The measures of the interior angles of a triangle always add up to 180°. For example, if we know that a triangle has interior angles of 37° and 56°, we know that the third interior angle is 87°.

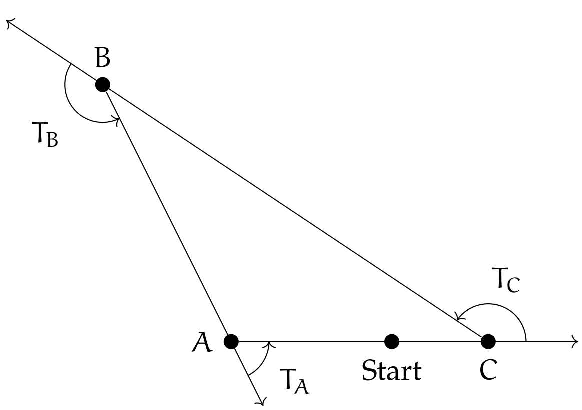
Exercise 1 Missing Angle

One interior angle of a triangle is 92°. The second angle is 42°. What is the measure of the third interior angle?

Working Space

Answer on Page 7

How can you know that the sum of the interior angles is 180°? Imagine that you started on the edge of a triangle and walked all the way around to where you started. (going counter-clockwise.) You would turn three times to the left:



After these three turns, you would be facing the same direction that you started in. Thus, $T_A + T_B + T_C = 360°$. The measures of the interior angles are a, b, and c. Notice that a and T_A are supplementary. So we know that:

- $T_A = 180 - a$
- $T_B = 180 - b$
- $T_C = 180 - c$

So we can rewrite the equation above as

$$(180 - a) + (180 - b) + (180 - c) = 360°$$

Which simplifies to:

$$540° - (a + b + c) = 360°$$

Subtracting both sides from 540:

$$a + b + c = 180°$$

Exercise 2 Interior Angles of a Quadrilateral

Any four-sided polygon is a *quadrilateral*. Using the same "walk around the edge" logic, what is the sum of the interior angles of any quadrilateral?

Working Space

Answer on Page 7

This is a draft chapter from the Kontinua Project. Please see our website (`https://kontinua.org/`) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 4)

$180° - (92° + 42°) = 46°$

Answer to Exercise 2 (on page 5)

$360°$

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