Chapter 1

Solving Quadratics

A quadratic function has three terms: ax^2+bx+c. a, b, and c are known as the *coeffcients*. The coefficients can be any constant, except that a can never be zero. (If a is zero, it is a linear function, not a quadratic.)

When you have an equation with a quadratic function on one side and a zero on the other, you have a quadratic equation. For example:

$$72x^2 - 12x + 1.2 = 0$$

How can you find the values of x that will make this equation true?

You can always reduce a quadratic equation so that the first coefficient is 1, so that your equation looks like this:

$$x^2 + bx + c = 0$$

For example, if you are asked to solve $4x^2 + 8x - 19 = -2x^2 - 7$

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$$6x^2 + 8x - 12 = 0$$

$$x^2 + \frac{4}{3}x - 2 = 0$$

Here, $b = \frac{4}{3}$ and $c = -2$.

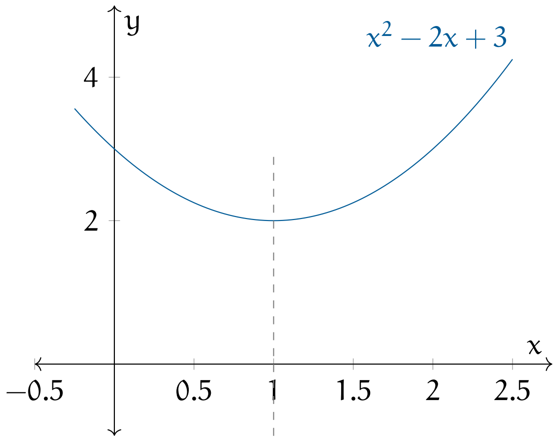
$x^2 + bx + c = 0$ when

$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2}$$

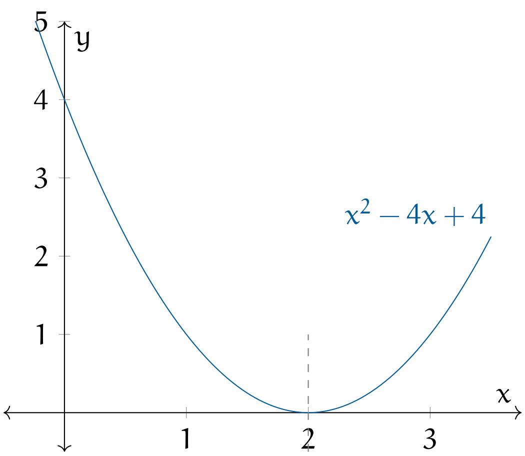
What does this mean?

For any b and c, the graph of $x^2 + bx + c$ is a parabola that goes up on each end. Its low point is at $x = -\frac{b}{2}$.

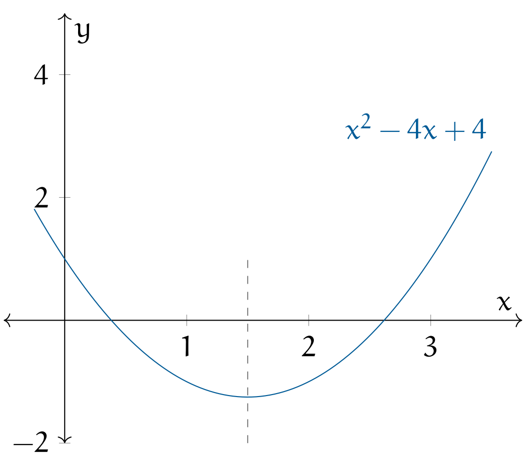
If there are no real roots ($b^2 - 4c < 0$), which means the parabola never gets low enough to cross the x-axis:



If there is one real root ($b^2 - 4c = 0$), it means that the parabola just touches the x-axis.



If there are two real roots ($b^2 - 4c > 0$), it means that the parabola crosses the x-axis twice as it dips below and then returns:



Exercise 1 Roots of a Quadratic

Working Space

In the last chapter, you found that the function for the height of your flying hammer is:

$$p = -\frac{1}{2}9.8t^2 + 12t + 2$$

At what time will the hammer hit the ground?

Answer on Page 5

1.1 The Traditional Quadratic Formula

If the last explanation was a little tricky to understand the quadratic formula is a nifty tool.

The Quadratic Formula

$ax^2 + bx + c = 0$ when

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is a draft chapter from the Kontinua Project. Please see our website (`https://kontinua.org/`) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 3)

For what t is $-4.9t^2 + 12t + 2 = 0$? Start by dividing both sides of the equation by -4.9.

$$t^2 - 2.45t - 0.408 = 0$$

The roots of this are at

$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2} = -\frac{-2.45}{2} \pm \frac{\sqrt{(-2.45)^2 - 4(-0.408)}}{2} = 1.22 \pm 1.36$$

We only care about the root after we release the hammer $(t > 0)$.

$1.22 + 1.36 = 2.58$ seconds after releasing the hammer, it will hit the ground.