

## CHAPTER 1

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# Introduction to Polynomials

Watch Khan Academy's **Polynomials intro** video at <https://youtu.be/Vm7H0VT1Ico>

A *monomial* is the product of a number and a variable raised to a non-negative (but possibly zero) integer power. Here are some examples of monomials:

$$3x^2$$

$$\pi x^2$$

$$7x$$

$$-\frac{2}{3}x^{12}$$

$$-2x^{15}$$

$$(3.33)x^{100}$$

$$3$$

$$0$$

The exponent is called the *degree* of the monomial. For example,  $3x^{17}$  has degree 17,  $-7x$  has degree 1, and 3.2 has degree 0 (because you can think of it as  $(3.2)x^0$ ).. The exponent cannot be  $x$  (or any non-constant variable), as that becomes an exponential function rather than polynomial.<sup>1</sup>

The number in the product is called the *coefficient*. Example:  $3x^{17}$  has a coefficient of 3,  $-2x$  has a coefficient of -2, and  $(3.4)x^{1000}$  has a coefficient of 3.4.

A *polynomial* is the sum of one or more monomials. Here are some polynomials:

$$4x^2 + 9x + 3.9$$

$$\pi x^2 + \pi x + \pi$$

$$7x + 2$$

$$-2x^{10} + (3.4)x - 45x^{900} - 1$$

$$3.3$$

$$3x^{20}$$

We say that each monomial is a *term* of the polynomial.

$x^{-5} + 12$  is *not* a polynomial because the first term has a negative exponent.

$x^2 - 32x^{\frac{1}{2}} + x$  is *not* a polynomial because the second term has a non-integer exponent.

$\frac{x+2}{x^2+x+5}$  is *not* a polynomial because it is not just a sum of monomials.

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<sup>1</sup>A quadratic is a polynomial with a maximum degree of 2

**Exercise 1**      **Identifying Polynomials**

Circle only the polynomials.

Working Space

$$-2x^3 + \frac{1}{2}x + 3.9(4.5)x^2 + \pi x \quad 7$$

$$2x^{-10} + 4x - 1 \quad x^{\frac{2}{3}} \quad 3x^{20} + 2x^{19} - 5x^{18}$$

Answer on Page 5

We typically write a polynomial starting at the term with the highest degree and proceed in decreasing order to the term with the lowest degree:

$$2x^9 - 3x^7 + \frac{3}{4}x^3 + x^2 + \pi x - 9.3$$

This is known as *the standard form*. The first term of the standard form is called *the leading term*, and we often call the coefficient of the leading term *the leading coefficient*. We sometimes speak of the degree of the polynomial, which is just the degree of the leading term.

**Exercise 2**      **Standard of a Polynomial**

Write  $21x^2 - x^3 + \pi - 1000x$  in standard form. What is the degree of this polynomial? What is its leading coefficient?

Working Space

Answer on Page 5

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**Exercise 3      Evaluate a Polynomial**

Let  $y = x^3 - 3x^2 + 10x - 12$ . What is  $y$  when  $x$  is 4?

Working Space

Answer on Page 5

We would be remiss in our duties if we didn't mention one more thing about polynomials: Mathematicians have defined a polynomial to be a sum of a *finite* number of monomials.

It is certainly possible to have a sum of an infinite number of monomials like this:

$$1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{16}x^4 + \dots$$

This is an example of an *infinite series*, which we don't consider polynomials. Infinite series are interesting and useful, but we will not discuss them in detail until later in the course.

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*This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.*



# Answers to Exercises

## Answer to Exercise 1 (on page 2)

$$-2x^3 + \frac{1}{2}x + 3.9$$

$$(4.5)x^2 + \pi x$$

$$7$$

$$2x^{-10} + 4x - 1$$

$$x^{\frac{2}{3}}$$

$$3x^{20} + 2x^{19} - 5x^{18}$$

## Answer to Exercise 2 (on page 2)

Standard form would be  $-x^3 + 21x^2 - 1000x + \pi$ . The degree is 3. The leading coefficient is  $-1$

## Answer to Exercise 3 (on page 3)

$$4^3 - (3)(4^2) + (10)(4) - 12 = 64 - 48 + 40 - 12. \text{ So } y = 44$$





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